August 19, 2024

A METHODOLOGY FOR CUTTING A BUDGET

by

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The subject of this paper is a methodology that can be used to cut a budget. Its purpose is to show that one can employ a sound methodology rather than use a meat axe approach or throw up one's hands saying cutting the budget is too hard. The methodology can used as a starting point when actually cutting a budget. It employs economic, accounting, and operations research notions and consists of five steps as described below. The mathematics are in the appendix. All computations were done with the LibreOffice spreadsheet and Gauss-24, a product of Aptech, Inc. The budget I am using is the Corvallis, Oregon, 2024-2025 proposed budget.

Direct and Indirect Costs

The first thing that has to be done is separate the costs into direct and indirect costs. Direct costs are costs incurred to provide service directly to the citizen such as police services. Indirect costs are costs incurred to provide service to the direct cost centers. For example, the human resources department provides services to the police department, not the citizens.

The direct costs I used in this analysis came from the Davis, Hibbitts & Midghall, (DHM) Inc. City Priorities and Program Funding Survey of 2010 prepared for the City of Corvallis – which is now fourteen years old. It would have been better to use a more recent survey, but if a more recent survey exists, I couldn't find it. I used this survey because it reflects citizens' thinking at large, rather than my thinking which would have been the case had I separated the department expenditures in the proposed budget into direct and indirect costs. The direct costs, which DHM used are: police, fire, library, social services, parks & recreation, and the aquatic center. The aquatic center is part of the parks & recreation expenditures in the budget which had to be separated from parks & recreation in this analysis. Social services are part of the mayor and city council expenditures which also had to be separated.

All other costs are indirect costs, hence they are overhead and need to be allocated to the direct costs to determine the fully burdened cost of police, fire, library, social services, parks & recreation, and the aquatic center.

The costs incurred are the expenditures by department. The numbers came from the general fund overview and the departments at a glance sections of the budget.[1] The notion being to estimate the amount of money that each department actually spends – which becomes a vector of department expenditures.

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1 The money in the utility funds was not included because the utility operation is essentially independent of the rest of the budget.

Cobb-Douglas Utility Function

The second thing that has to be done is to develop a Cobb-Douglas utility function for the direct costs that can be used to allocate budget cuts so that the reduction in utility is minimized. There is sufficient information in the DHM Survey to develop such a function. The survey asked the respondents to allocate a fixed amount of money to the six direct costs. The averages of those numbers can be used to develop the utility function.

Cost Allocation

The third thing that has to be done is to allocate the indirect costs to the direct costs. Or, in other words, fully burden direct costs so direct costs reflect the total costs to deliver that service. A way to do this is to build an allocation matrix that reflects what a department contributes to all the departments, including itself. For example, the cost of the human resources department can be allocated to all departments by the fraction of the total number of full time equivalents in each department. The cost of the finance department can be allocated by the fraction of total expenditures each department spends.[2]

The allocation matrix is then pre-multiplied by the transpose of the department expenditure vector to get the transpose of the first allocation of costs vector. The first allocation does not allocate much of the indirect costs to the direct costs. Therefore the transpose of the first, and subsequent allocation vectors, have to be repeatedly multiplied the allocation matrix N times to get a final allocation that leaves a fraction of a dollar in each indirect cost account and fully burdens all direct costs. In this case N was twenty. A byproduct of this procedure is a “grand allocation matrix” that is the allocation matrix multiplied by itself N times.[3]

Budget Cuts

The fourth thing that has to be done is decide on the budget cut and allocate that cut to the fully burdened direct costs in a manner that makes the smallest reduction in the utility function. I decided to make a total budget cut of $1,000,000 because the total recurring revenue less recurring expenditures was slightly more positive than minus $1,000,000 for FY 25-26. The notion being that cutting the budget that much would allow the city council to avoid increasing the fees — that appear on our water bills — next January.

I randomly assigned cuts (totaling $1,000,000) to the six fully burdened direct costs a thousand times, computed the effect of each set of cuts on the utility function, and chose the set of cuts with the least negative change to the utility function.

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2 I could not find another metric which I could use to allocate all other indirect cost departments so I had to use the fraction of total expenditures for all departments other than human resources. I would like to have allocated the information technology costs by the number and sophistication of the computers each department had but that information is not in the budget.

3 The grand allocation matrix will be needed to allocate the cuts back to the original departments.

Back-Allocation of Budget Cuts

The fifth thing that has to be done is to back-allocate the cuts in the fully burdened direct costs back to the original direct and indirect cost centers using the grand allocation matrix so that specific cuts can be made to specific departments. When making cuts, particular attention needs to be paid to expenditures that are financed by “Special Revenue Funds” because these funds account for proceeds of specific revenue sources that are legally restricted to specific purposes. Cuts may have to be made to expenditures financed by the general fund.

The suggested cuts, in dollars, to the Corvallis 2024-2025 proposed budget are shown below.

Departments	Suggested cuts
Mayor & coun (-)	-70948.7
Social service	-31967.1
City manager	-70955.6
Com dev	-70955.6
Finance	-70955.6
Fire	-86202.8
Hum res	-73086.2
Inf tech	-73216.4
Library	-126945.6
Pks & rec. (-)	-135836.5
Osb Cen	-41019.8
Police	-573.4
Pubwks	-73668.3

APPENDIX

Cobb-Douglas Utility Function

The equation for a Cobb-Douglas utility function[4], U, is: $U = \alpha \prod x_i^{\beta(i)}$ where α and β(i) are constants, and x_i is the amount of money allocated to direct cost “i”. Setting U to 1 makes α vanish.

$\partial U / \partial x_i = \beta(i) / x_i$

Set $\partial U / \partial x_i = 0$ for all x_i.

4 A reference is: Berek, P. & K. Sysdaeter, Economists’ Mathematical Manual. Springer-Verlag, Heidelberg, Germany. P 132.

Set up the following equation in matrix format: $D\beta\text{-}0=\varepsilon$, where D is a diagonal matrix, the diagonal being $1/x_i$. β, 0, and ε are vectors. Solve the following equation: $(D\beta\text{-}0)^{\prime} (D\beta\text{-}0)=\varepsilon^{\prime}\varepsilon$ for β by minimizing $\varepsilon^{\prime}\varepsilon$ subject to $\sum\beta(i) = 1$ with a quadratic programming computer program[5].

Cost Allocation

The allocation matrix is a square matrix in which the cells in each row are each departments' fractional contributions to all the departments, including itself. Direct service departments do not make any contributions to any other departments, hence the row entry for direct service departments are a one in the diagonal cell and a zero in all other cells in that row. The equation for allocating the indirect costs to direct service costs is: $\gamma(0)^{\prime} A = \gamma(1)^{\prime}$, where γ(0) is the original department expenditure vector, A is the allocation matrix, and γ(1) is the department expenditure vector after the first allocation. Multiply $\gamma(1)^{\prime}$ by A to get $\gamma(2)^{\prime}$ and so on to get $\gamma(N)^{\prime}$ – the final allocation, where N is, in this case, twenty. As mentioned above, a byproduct of this procedure is a "grand allocation matrix", Ĝ, that is the allocation matrix multiplied by itself N times. $\gamma(0)^{\prime} \hat{G} = \gamma(N)^{\prime}$.

Budget Cuts

I assigned random cuts – totaling $1,000,000 -- to the fully burdened direct costs using uniform continuous distributions and calculated the change in the utility function with its differential: $dU = \sum \beta(i)\, dx_i / x_i$ where dx_i is the cut assigned to cost "¡" and x_i is fully burdened cost "¡". I did this 1000 times. The set of cuts with the least negative differential was chosen as "the" set of cuts.

Back-Allocation of Budget Cuts

To back-allocate the fully burdened direct costs to the original direct and indirect costs I used the equation $\hat{C}^{\prime} \hat{G} = Ç^{\prime}$ where Ĉ is the vector of original direct and indirect costs, and Ç is the vector of fully burdened direct costs[6]. Rearranging this equation and applying the notion of least squares results in $(\hat{G}^{\prime} \hat{C} - Ç)^{\prime} (\hat{G}^{\prime} \hat{C} - Ç) = \varepsilon^{\prime}\varepsilon$ which can be solved for Ĉ by minimizing $\varepsilon^{\prime}\varepsilon$ subject to the sum of the cuts in the fully burdened direct costs equal to the sum of the cuts in the original direct and indirect costs with a quadratic programming computer program.

5 A reference for quadratic programming is Taha, H.A. Operations Research, An Introduction. Collier Macmillan Publishing Company. 1971, Page 639.

6 The vector of fully burdened direct costs contains entries of zero as indirect cost place holders.